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Automotive Doors as Loudspeaker Enclosures Modeling Considerations

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ABSTRACT

A study of automotive doors as loudspeaker enclosures was previously presented [1] [2]. Considerations for modeling the mechanical and acoustical behavior of automotive doors are now presented. Theoretical mathematical models and computer modeling considerations for modeling the mechanical-acoustic behavior of automotive doors as loudspeaker enclosures are presented. The goal for the models is to predict the mechanical dynamic behavior and consequently the impedance and frequency response of the loudspeaker door enclosure. Modeled results would then be used to compare to impedance and frequency response measurements of several doors. This modeling would further investigate a methodology for quantifying door enclosures and refines the criteria for qualifying automotive doors as loudspeaker enclosures.

1. REVIEW

At the end of the previous work, [1] and [2], we came to the conclusion that a need exists for further investigation into the source of the different non-ideal behaviors of loudspeakers in doors.

1.1. Door Structure

The focus of this work, again, is using an automotive door as a loudspeaker enclosure. It is known from [1] and [2] that the sheet metal of the door rarely forms a sealed enclosure and therefore leaves the interior volume open to the door's trim. The trim sometimes becomes a vital part of the enclosure system. We have seen that the linking of the inner door volume to the outer volume created by the door trim could be beneficial, if it can be done in a way that is controlled enough not to excite the door trim to the point of creating extraneous noise.

Sources of enclosure volume variability and extraneous noise include the following:

- Window motors, windows, window tracks, side impact beams, and door release mechanisms

- Thin trim materials, poor trim seams, switch panels, lighting assemblies, map pockets, and speaker grills
- Lack of a sufficient quantity of inner sheet metal to create a baffle or complete enclosure
- Trim that is required to function as one wall of the enclosure; the coupling of the sheet metal volume to the trim is critical, but variable
- Trim attachment methods that are variable and unstable over time

Figure 1: Automotive Door Example w/o trim

Figure 2: Automotive Door Trim Exterior

Figure 3: Interior Automotive Door Trim

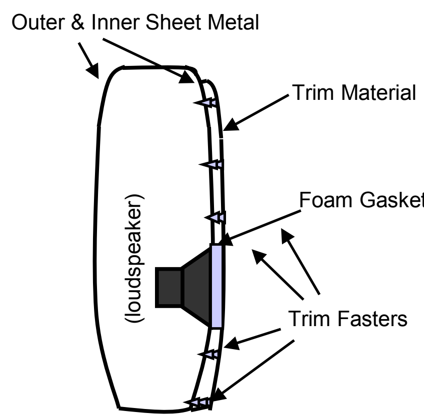

Figure 4: Loudspeaker and Door Enclosure System

1.2. Not Simple Ideal Box Theory

One of the initial targets of the study was to compare the door enclosure to typical box enclosures and qualify them against ideal box design parameters. Impedance data and frequency response measurements were made for the purpose of comparing to ideal box theory [1] [2]. The conclusions of this are as follows:

- Impedance measurements with & without trim show that none of the door enclosures follow simple ideal box theory
- Resonance decreases with trim in place
- Simple ideal box theory tells us that the resonance frequency and Q_t should increase; this does not always happen, but because we are applying non-

linear data to a linear model, the values of the enclosure parameters may be meaningless

- The data does not appear to adhere to the linear model, which is a second-order system; we need to apply another analysis; the door/enclosure system is more complex than a second-order system

- Important Note: A drop in resonance indicates, at the very least, the influence of another system created by the trim, with increased efficiency as a potential result

1.3. Impedance Curve Types

Looking at the impedance data in Appendix 1, four types of impedance curves emerge:

- *Type I* = a minimal change from the free-air impedance curve of the loudspeaker

- *Type II* = a skewing of the impedance curve that becomes asymptotic to the free-air impedance curve below the resonance frequency

- *Type III* = a complete shift down of the impedance curve and therefore the resonance frequency

- *Type IV* = an impedance curve with a double maximum that resembles a ported enclosure

The door enclosures can be categorized by their impedance curves with and without the trim attached.

For doors classified as *Type III* (complete shift down of the impedance curve), there appears to be an external system influence occurring that creates the apparent increase in moving mass. However, calculations show that *Type III* doors exhibited no more than a 3% increase in moving mass. The *Type II/II* doors exhibited up to a 25% increase in moving mass. The skewing of the impedance curve for doors classified as *Type II* may imply there is a greater influence from the door itself. The phenomenon of the resonance frequency shifting down and the impedance curve no longer staying asymptotic to the free-air impedance curve (*Type III/ Type III*) is unexplained.

It is not possible from the data collected to determine the exact cause of the resonance shift. The door enclosures are presumed to be lossy enclosures with less than rigid baffles. This may contribute to the apparent increase in moving mass and lower resonance frequency. This lossy characteristic of the door enclosures is also a source of uncontrolled behavior that must be understood and manipulated. The lowering of the resonance frequency can obviously benefit the low frequency output, but at the same time the unknown attributes it brings is cause for precaution [1] [2].

1.4. Frequency Response Changes with Trim

- In nearly every case, the addition of the door trim increased the low frequency output of the door enclosure system

- Door systems classified as *Type II* with their trim attached consistently showed a significant increase in low frequency output and most notably if the door was classified as a *Type II/II* (a skewing of the impedance curve that becomes asymptotic to the free-air impedance curve below the resonance frequency**)**

- The <u>trim</u> was instrumental in increasing the low frequency output for *Type II/II* doors

- For *Type III/III* or *Type II/III* doors, the <u>door</u> was governing the respective behavior and <u>not the trim</u>

Door	Imp Types (No Trim/Trim)	Freq. Response Change No Trim -> Trim
1	II / II	50 – 150 Hz, +3 dB
2	I / II	40 – 50 Hz, +2 dB
3	II / II	20 – 1k Hz, +5 dB
4	III / III	None
5	III / II	20 – 1k Hz, +4 dB
6	II / II	50 – 150, +3 dB
7	II / IV	125 – 200, - 3 dB
8	II / III	20 – 600, + 3 dB

Table 1: No Trim / Trim Impedance Type & Frequency Response

2. MODELING GOALS & METHODS

The primary goal of the modeling is to replicate the impedance and frequency response. The first modeling attempt will be Door 5 from the previous work. It is a mid-sized SUV door, characterized as a *Type III* without trim and a *Type II* with trim (*Type III/II*). It exhibits a 4 dB boost from 20 Hz – 1 kHz (effectively the passband of the 16 cm loudspeaker mounted in the door) caused by the door trim. The impedance curve for the loudspeaker mounted in the door exhibited a complete shift down in resonance frequency as compared to the free-air impedance curve. It also exhibited a double maximum in its impedance curve. When the trim was added, the impedance curve became a *Type II* (a skewing of the curve that becomes asymptotic to the free-air curve). The model will attempt to replicate these attributes

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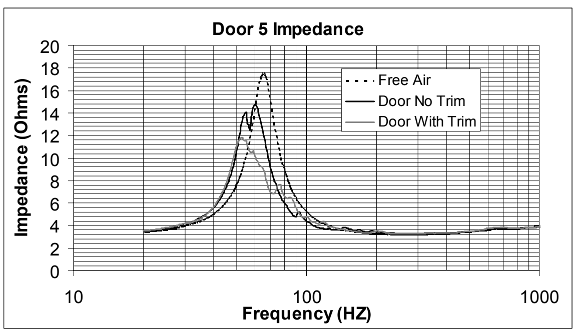


Figure 5: Door 5 Impedance Curves

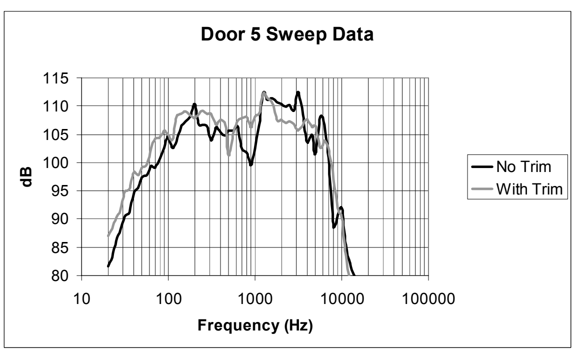


Figure 6: Door 5 Frequency Response Curves

Door 5	Free Air	Door	With Trim
F	65.48	60.40	52.60
Q_m	4.41	3.98	2.18
Q_e	0.99	1.13	0.83
Q_t	0.81	0.88	0.60
V_D (liter)	0.04		
M_{ms}	10.08	12.56	
α		0.05	-0.33

Table 2: Door 5, Medium-Sized SUV, Parameters

Simulation Method

The goal of the simulation is to develop a model that will explain the behavior. An empirical approach is taken to determine the correct model and test parameters. It will be more difficult to predict future door geometries using this method unless obvious relationships can be drawn from the test parameters and the actual physical case. Attempting to fit all sets of impedance data collected so far will test the simulation model.

Finite Element / Boundary Element

The FEM/BEM model should better predict the behavior of any given physical case if enough detail can be modeled and the boundary conditions can be defined so as to provide results within the frequency range of interest. In this case, the frequency range of interest is 20 – 500 Hz.

3. SIMULATION MODEL DESIGN

The impedance measurements were taken at low voltage levels that were assumed to be in the linear operating range of the loudspeaker. All observations were made at low frequencies. The first approach is a lumped-parameter model with the loudspeaker modeled as a spring-mass-damper system. The first assumption of the simulation is that the second peak, or any perturbing of the resonance, is due to a second system being driven by the first system. There is also some loading in the second system that affects how well or how badly the first system can drive the second system. The second system is assumed to be the total mass of the loudspeaker assembly and other parts (motors, etc.) that are mounted to the door's sheet metal close by the speaker. This second system also takes into account the potential flexibility of the sheet metal. The affect of adding the trim is modeled as an additional spring and damper.

3.1. Transducer

The loudspeaker system is shown schematically in Fig. 7 as a spring-mass-damper system driven by a voice coil:

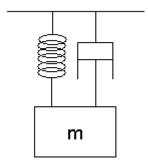


Figure 7: Loudspeaker System

The equation defining this system is [see Appendix 2]:

$$L_e \frac{dI}{dt} + R_e I = V - Bl \frac{dx}{dt} \tag{2}$$

Transducer Parameters:

M_{ms}	lumped mechanical moving mass including air load
R_{ms}	lumped mechanical resistance
K_{ms}	lumped mechanical springiness
C_{ms}	lumped mechanical compliance (1/Kms)
Bl	force factor
R_e	DC resistance of voice coil
L_e	inductance of voice coil
S_d	effective piston area

3.2. Basket / Mounting Point

In the linear, lumped parameter model for the loudspeaker, the assumption is made that the spring and damper are attached to a mechanical ground on the ends opposite the mass element. When the loudspeaker is mounted in such a fashion that the basket is rigidly attached to a rigid mounting surface, this is a close approximation. However, in a car door, the metal mounting points can flex appreciably, and thus a new, more complex model is necessary. In keeping with the theme of simplicity, the spring and damper will be attached to a mass element, with the mass element being the effective mass of the transducer and door mounting area, and the spring being the effective springiness of the mounting point. These will be attached to mechanical ground on their opposite ends. The mechanical schematic is shown below in Fig. 8:

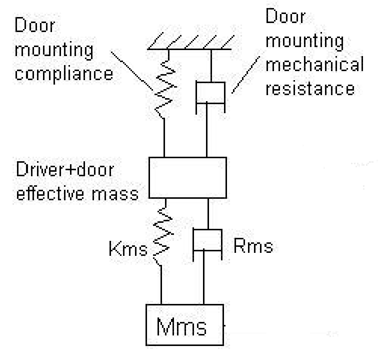


Figure 8: Mounting Point Model

The equation defining this system is [see Appendix 2]:

$$M_{ms} \frac{d^2 x_1}{dt^2} + \left(S_d^2 R_{door} + R_{ms}\right) \frac{dx_1}{dt} + \left(S_d^2 K_{door} + K_{ms}\right) x_1 - R_{ms} \frac{dx_2}{dt} - K_{ms} x_2 = BlI(t) \tag{4}$$

Mounting Point Parameters:

M_{mount}	lumped mechanical moving mass of transducer/mounting point (force/acceleration = kg)
R_{mount}	lumped mechanical resistance of mounting point (force/velocity = N/(m/s))
K_{mount}	lumped mechanical springiness of mounting point (force/displacement = N/m)

3.3. Door As Enclosure

The conventional model for the enclosure uses only the acoustic compliance of the enclosure, assuming no losses (panels are rigid). A car door, however, is a very leaky environment, and thus an acoustic resistance element will be included in the model. The new model is shown below in Fig. 9.

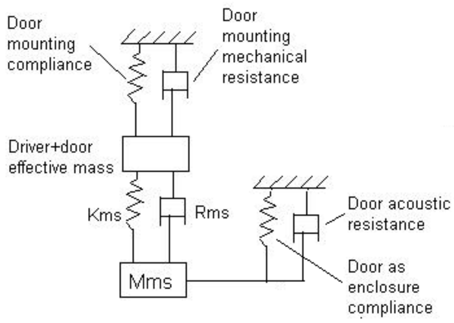


Figure 9: Mounting Point and Enclosure Model

The equation defining this system is [see Appendix 2]:

$$-R_{ms}\frac{dx_1}{dt} - K_{ms}x_1 + M_{mount}\frac{d^2x_2}{dt^2} + \left(R_{mount}+R_{ms}\right)\frac{dx_2}{dt} + \left(K_{mount}+K_{ms}\right)x_2 = 0 \qquad (6)$$

Enclosure Parameters:

R_{door} lumped acoustical resistance of mounting point (Press./Vol. velocity = Ns/m^5)

K_{door} lumped acoustical springiness of mounting point (Press./Vol. displacement = N/m^5)

3.4. Transfer Function and Impedance Equation

The derivation of the transfer functions and impedance equation is given in Appendix 2. These equations completely describe the behavior of the loudspeaker-door enclosure system. The mechanical system diagram with assigned displacements (x_1 & x_2) and labeled parameters is shown below in Fig. 10.

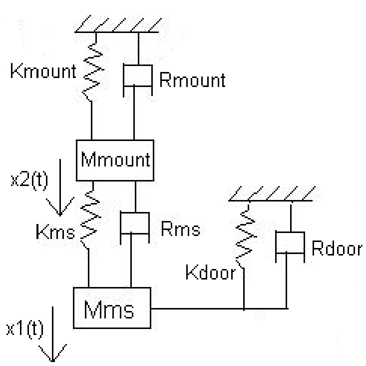


Figure 10: Loudspeaker-Door Enclosure System

The transfer function equations, (8) and (9), for x_1 and x_2 and the impedance equation, (11), are given in Appendix 2.

The transfer functions and the impedance equation look very similar to those for a ported enclosure. The difference lies in the added mechanical resistance of the door parameter. For a ported enclosure, another mass element is attached to the loudspeaker's spring-mass-damper system, but the acoustical resistance of the port is left off. The mechanical resistance of the door will be an additional parameter, which, upon variation, can change not just the quantitative but also the qualitative behavior of the impedance curve. With the mechanical resistance extremely low, the behavior will mimic that of another attached spring-mass system and the characteristic dual impedance spikes will be seen. With the resistance extremely high, the term with R_{mount} in its denominator will become trivially small, and the impedance curve will resemble that of a sealed box or free-air loudspeaker. An additional note is the acoustic resistance of the door. It acts in conjunction with the mechanical resistance of the loudspeaker, and therefore, the greater the acoustic resistance, the lower the impedance peak will be at resonance. A very leaky door may flatten the curve considerably.

4. FEM / BEM MODEL DESIGN

The mesh can be created from a given CAD model of a door or from measurements of an actual door. Measurements from an actual door were used for the first simulation trial. However, any detail that may be needed to refine the FEM/BEM model can be obtained from the CAD model and added to the FEM/BEM model geometry.

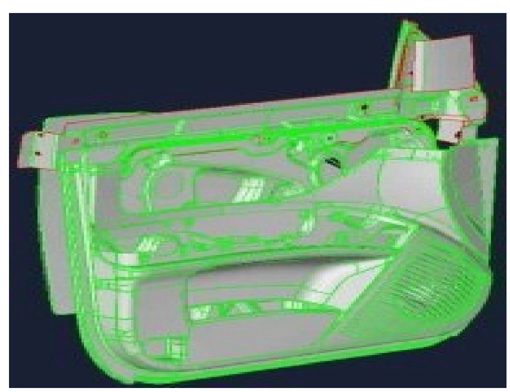


Figure 11: Door CAD Model

Figure 12: Loudspeaker CAD Model

The geometry (Fig. 13) created from measuring the dimensions of the door and loudspeaker is crude as a physical representation. This has an affect on the overall pressure field when the door panels vibrate. Rather than refining the geometry to better represent the CAD model, imposing better conditions and clamping the nodes that correspond to screw attachment points in the door panel will increase the accuracy of the results and reduce the total model size.

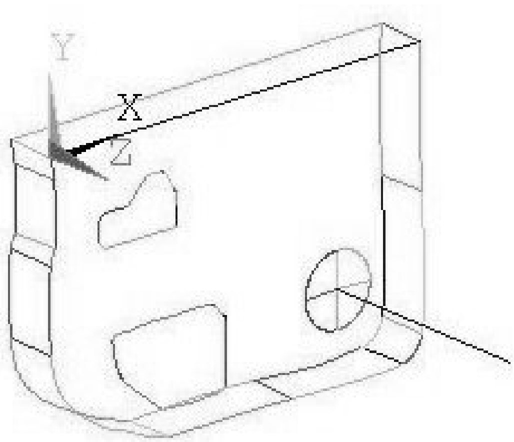


Figure 13: FEM Model

The line in the speaker axis, illustrated in Fig. 13 above, is a reference to help locate a node in the model at 0.5 m from the speaker, corresponding to the point at which the acoustic measurements were made [1] [2].

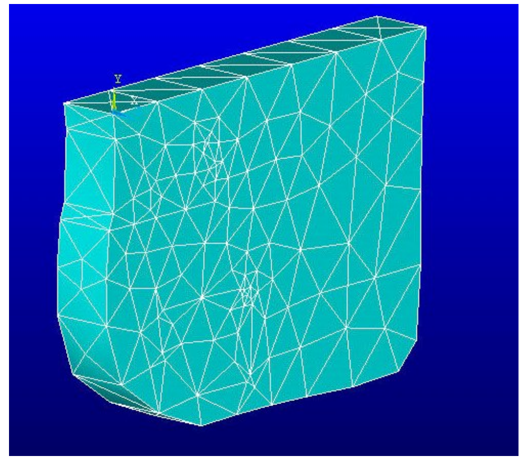


Figure 14: FEM Model Door Mesh

The air is modeled with a 3D linear fluid element and the door/door panel assembly with a linear shell element. A half-sphere of approximately 1.5 m is created, as illustrated in Figs. 15-16. The outer boundary is open and exhibits a “free-field” condition.

The essence of the FEM/BEM model will be the coupling of the opening in the door to the rest of the fluid, i.e. the air inside the door to the air outside the door.

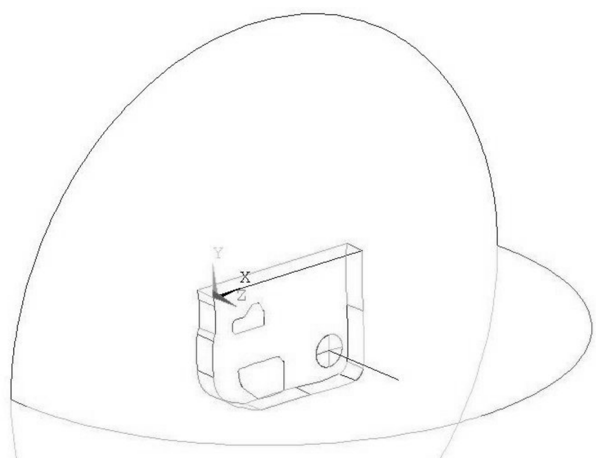


Figure 15: BEM Model

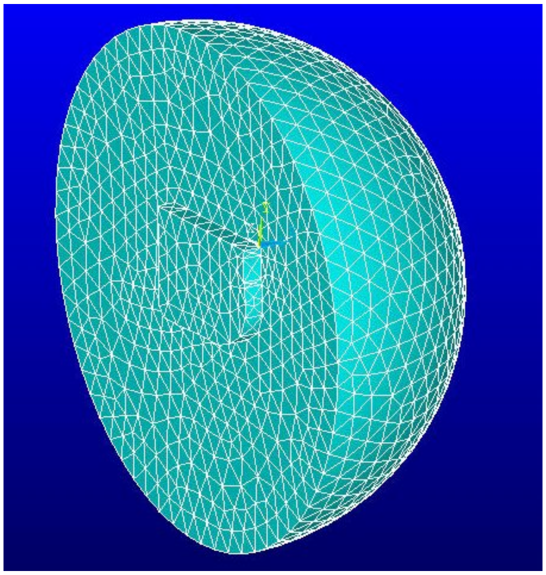


Figure 16: BEM Model Mesh

First, a modal analysis will be run from the FEM model. Second, using the loudspeaker as the source, a forced response will be run. The results of this will allow acoustic radiation to be calculated using the BEM model. The frequency response data will be obtained from the acoustic radiation solution. Impedance will be derived from the forced response solution, with impedance as a ratio of force/velocity. Figs. 17-20 show examples of pressure response contours expected from the model. The examples illustrate 300 Hz.

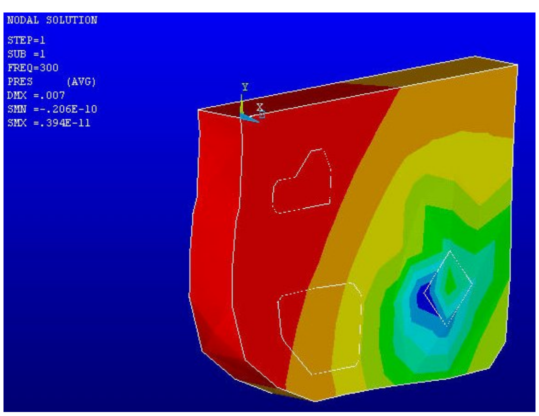


Figure 17: Real Pressure Contours (Door)

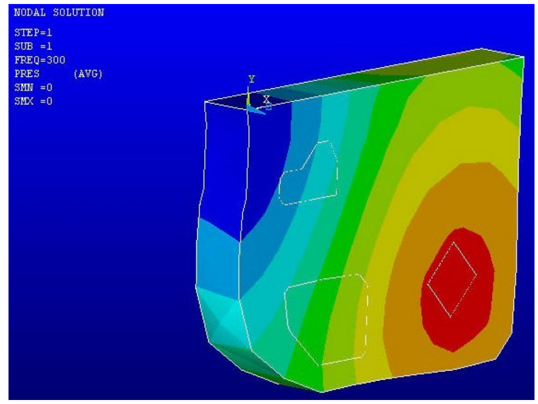


Figure 18: Imaginary Pressure Contours (Door)

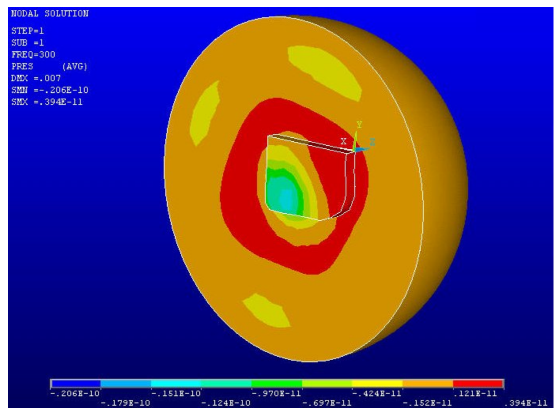


Figure 19: Real Pressure Contours (Half Space)

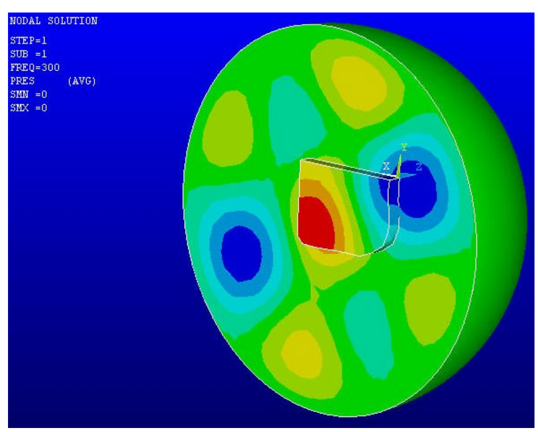


Figure 20: Imaginary Pressure Contours (Half Space)

5. SIMULATION RESULTS

Figs. 21-23 show some of the results of the simulation model for Door 5's loudspeaker/enclosure system. Matches to the measured data were found using the simulation model.

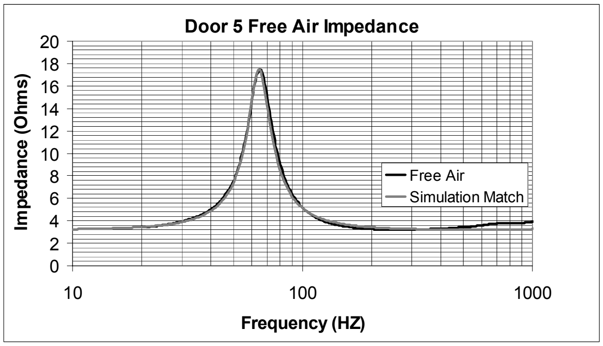


Figure 21: Door 5 (Free-Air) Imp. Simulation Match

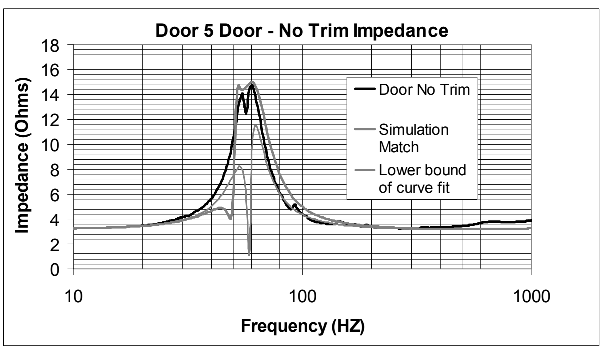


Figure 22: Door 5 (No Trim) Imp. Simulation Match

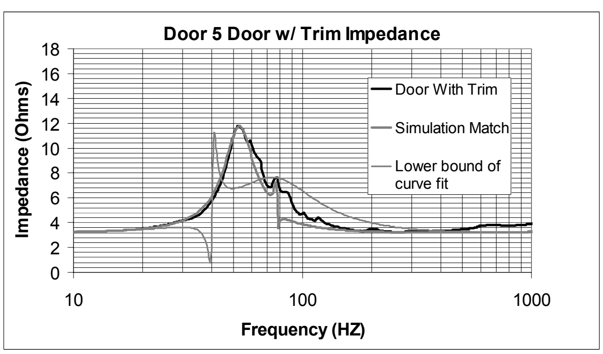


Figure 23: Door 5 (With Trim) Imp. Simulation Match

	Free Air	Door (No Trim)		Door (w/ Trim)	
	Match	Lower Bound	Match	Lower Bound	Match
Mms	22.5	27	23.5	16	35
Rms	2	2	2	2	2
Cms	268	268	268	268	268
Bl	5.35	5.35	5.35	5.35	5.35
Re	3.2	3.2	3.2	3.2	3.2
Le	0.13	0.13	0.13	0.13	0.13
Sd	153	153	153	153	153
Kdoor	0	0	0	20000	1000
Rdoor	0	0	3300	20000	5000
Mmount	1000000	1.1	1.1	0.423	0.649
Rmount	1000000	4	24	7	7
Kmount	1000000	150000	100000	20000	150000

Table 3: Parameters for Simulation Model

Through investigation and use of the model, it can be seen that the parameters for the simulation model can be used to control the shape of the impedance curve. The majority of control for the second resonances that appear in the measurements is in the damped spring-mass system that represents the mounting of the loudspeaker to the door, i.e. the "mount" parameters. The "door" parameters have amplitude control of the primary resonance and the ability to skew the resonance peak, but it keeps it asymptotic to the other curves. When said that these parameters control the impedance curve, it is meant these are the parameters that were originally thought to have influence over the shape of the impedance curves as well as the frequency response curves and therefore the bass response from the door system. As seen in Table 3, these parameters can be used to control the shape of the impedance curve in a way that corresponds to our measurement experience. M_{ms}, mentioned below, is not the only parameter that controls the resonance of a system. Also, during correlation work C_{ms} and M_{ms} will be watched more closely to determine if the modeling parameters are realistic. This would in turn mean the model has a valid physical use.

R_{door} – controls the amplitude of the primary resonance

M_{ms} – controls the frequency of the primary resonance (M_{ms} will be in combination with C_{ms})

M_{mount} & K_{mount} – controls the frequency of the secondary resonance

K_{mount} – also influences the amplitude of the secondary resonance

R_{mount} – controls the amplitude of the secondary resonance and the coupling/damping of the second resonance with respect to the first resonance

K_{door} – showed no influence on the Door-No Trim system, but showed an effect on the slope of the curve after the primary resonance for the Door w/Trim system; the slope increases or decreases proportionately with K_{door}

Figs. 24-25 below illustrate the control the above-mentioned parameters have on the simulation model.

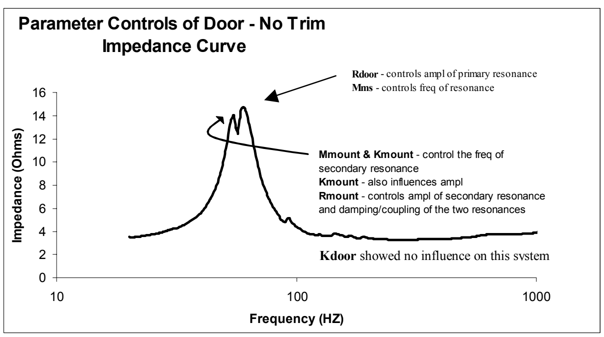


Figure 24: Parameter Controls of Door-NoTrim Impedance Curve

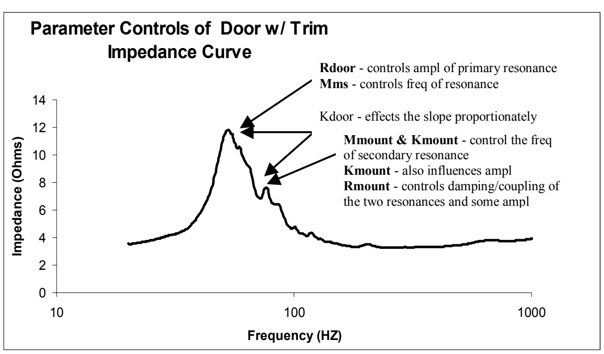


Figure 25: Parameter Controls of Door w/Trim Impedance Curve

The implications of the above parameter controls are, partially, the assumptions made when the model was designed.

- If the mount of the loudspeaker to the door were rigid, the second resonances would disappear.

Less intuitive for us was the following:

- If the stiffness of the door were allowed to increase, there would be less skewing of the impedance curve.

The model appears to be useful for, at the very least, describing the behavior of this particular door, Door 5 (*Type III/II*), and more so a tool for commenting on its design – and potentially controlling its design. Testing to determine if the modeling parameters make physical sense is necessary and will require further correlative measurements. The possibility of extension of this model to other loudspeaker/door systems will come from modeling the other systems that have been measured.

6. NEXT STEPS

Further correlation is the next step for the impedance simulation model. Further refinement of the model will be a part of that process. Improvements can be made in modeling the controlling terms of the system. For example, adding parallel resistance or other controls to better model the transition between the primary and secondary systems. A frequency response simulation will also follow.

Further work with the FEM/BEM model will be done. This will include parameter optimization and refinement of the model with the possibility of adding any necessary entities.

The purpose of the two modeling approaches, simulation and FEM/BEM modeling, is to allow the two approaches to guide each other as each approaches an optimal model for all of the data in hopes of a better understanding of the model as a whole. At some point, before any model reaches optimization, it could be abandoned based on accuracy and ease of use.

Throughout the modeling process there may exist a need for a better mathematical approximation of any given aspect of the system.

The modeling will continue after submission of the manuscript with more complete results and modeling approaches to be presented during the 118th AES Conference in Barcelona, Spain. Following that, results and refinements will be reported at a subsequent AES conference.

7. ACKNOWLEDGMENTS

The authors would like to acknowledge and thank Nolen Ryba and Gabriel Ruiz for their invaluable assistance in creating the simulations and FEA models, respectively. The current work would not have been possible without their assistance.

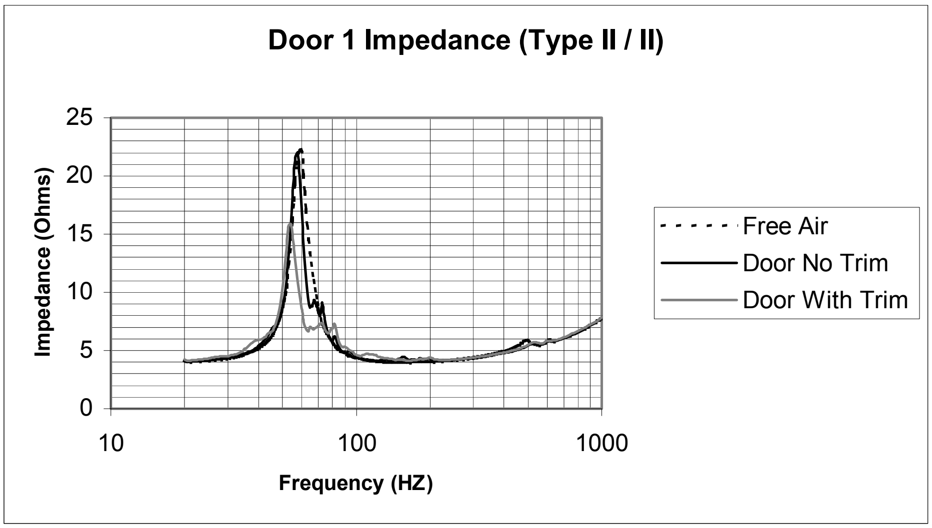
8. REFERENCES

[1] Roger Shively, Josh King, "Automotive Doors as Loudspeaker Enclosures," presented at the AES114th Convention, Amsterdam, The Netherlands, 2003 March 22-25, Preprint 5752.

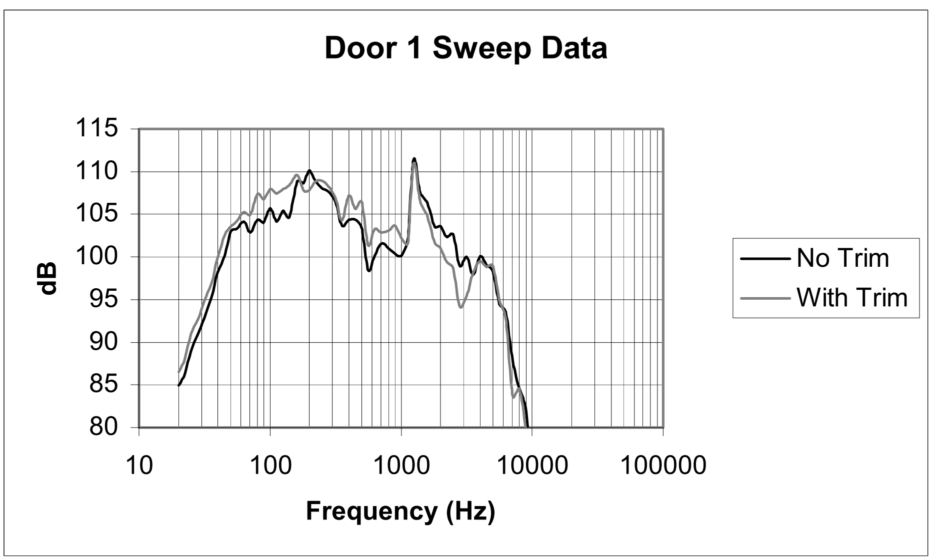
[2] Roger Shively, Josh King, "Update to Automotive Doors as Loudspeaker Enclosures," presented at the AES116th Convention, Berlin, Germany, 2004 May 8-11, Preprint 6002.

9. APPENDIX 1 DOOR 1

Door 1	Free Air	Door	With Trim
F	59.03	57.68	53.83
Q_m	7.56	6.18	8.50
Q_e	1.63	1.39	2.98
Q_t	1.34	1.13	2.21
V_D (liter)	0.07		
α		-0.17	0.67



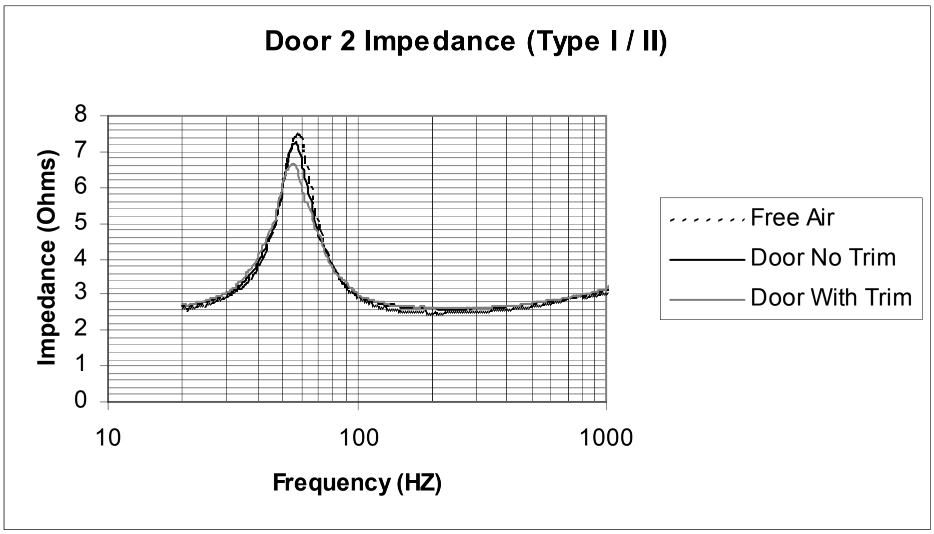
Type II Door Impedance: Skewing of the curve becoming asymptotic to the free air curves below resonance



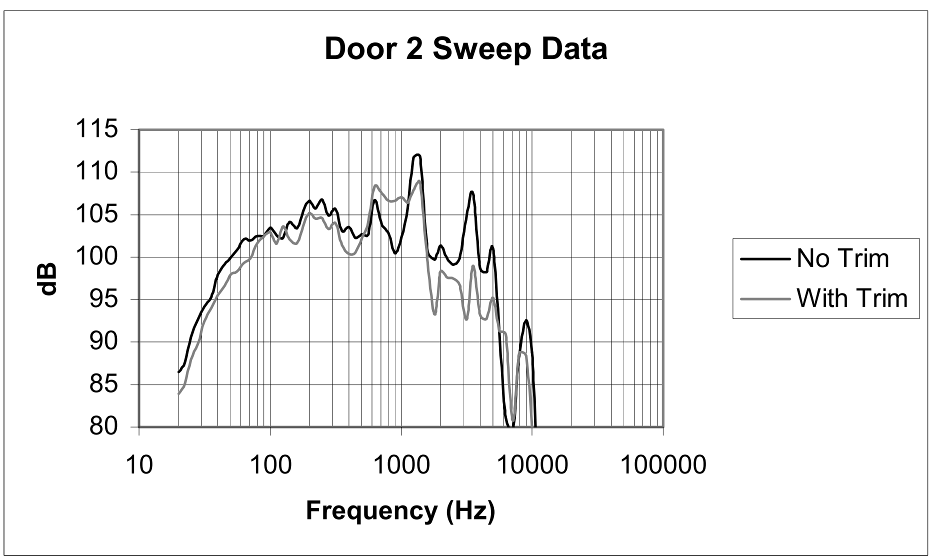
APPENDIX 1 (CONT.) DOOR 2

Door Type: Sports Coupe

Door 2	Free Air	Door	With Trim
F	57.03	56.38	55.73
Q_m	3.28	3.04	2.56
Q_e	1.62	1.68	1.65
Q_t	1.09	1.08	1.00
V_D (liter)	0.05		
α		0.02	-0.01



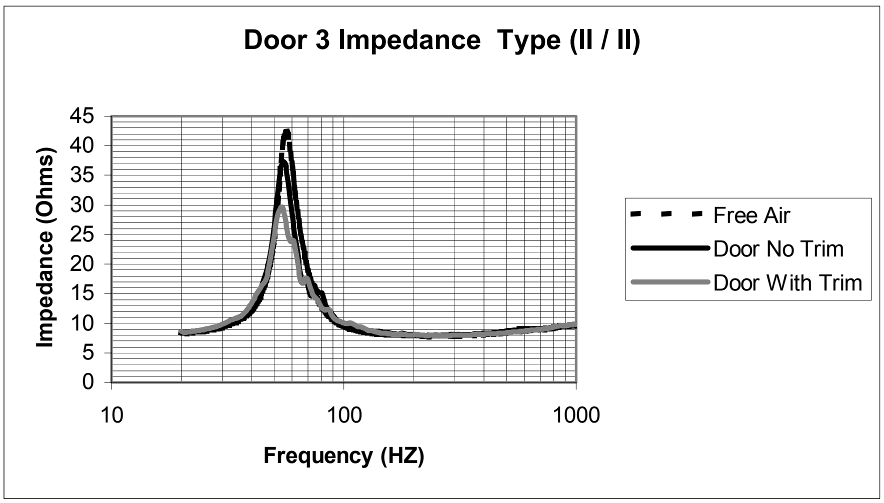
Type I Door Impedance: Minimal change from free air



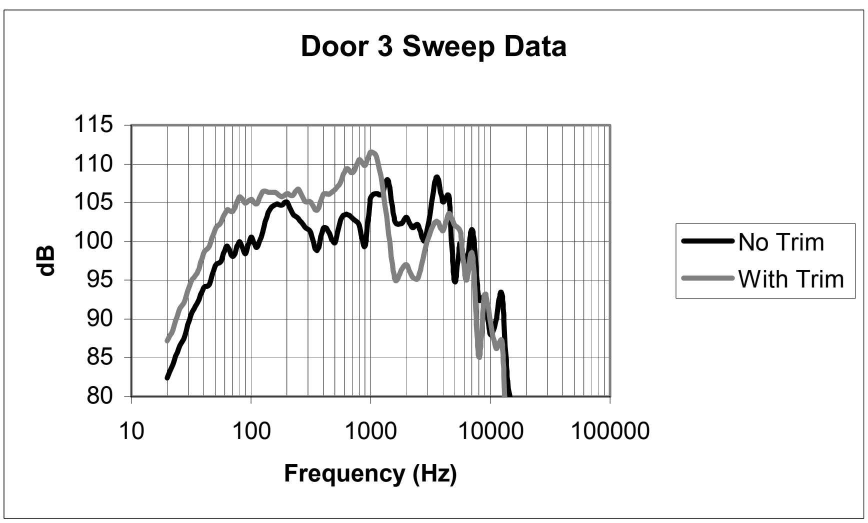
APPENDIX 1 (CONT.) DOOR 3

Door Type: Large Sedan

Door 3	Free Air	Door	With Trim
F	57.03	56.38	55.73
Q_m	3.28	3.04	2.56
Q_e	1.62	1.68	1.65
Q_t	1.09	1.08	1.00
V_D (liter)	0.08		
M_{ms}	15.03	19.00	
α		0.04	-0.04



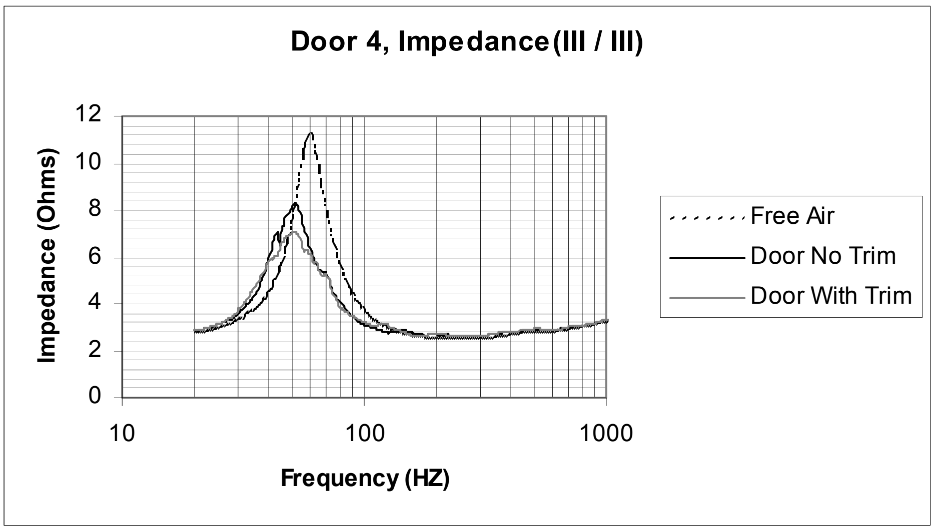
Type II Door Impedance: Skewing of the curve becoming asymptotic to the free air curves below resonance



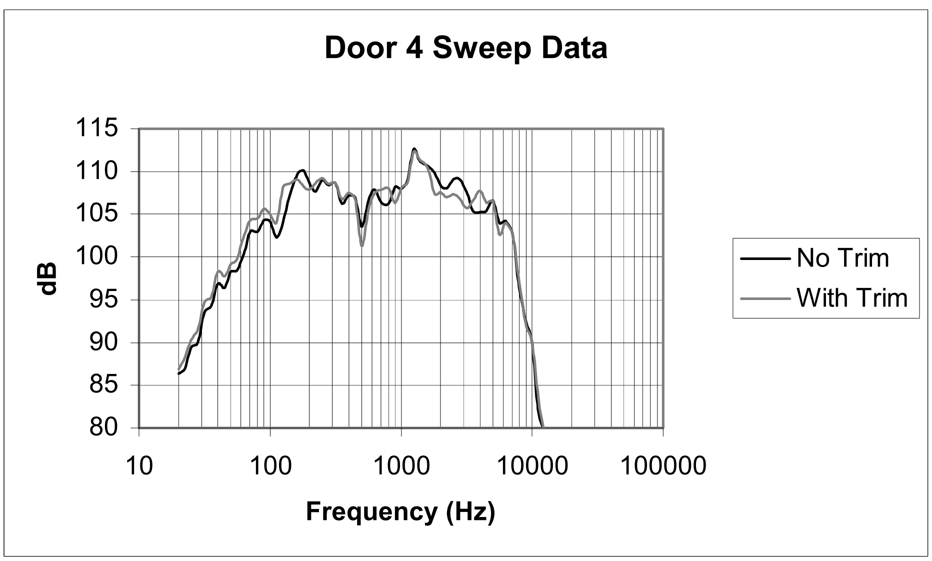
APPENDIX 1 (CONT.) DOOR 4

Door Type: Medium-Sized SUV

Door 4	Free Air	Door	With Trim
F	59.03	52.00	52.00
Q_m	3.31	2.49	2.03
Q_e	0.95	1.16	1.20
Q_t	0.74	0.79	0.75
V_D (liter)	0.11		
M_{ms}	12.00	12.43	
α		0.07	0.11



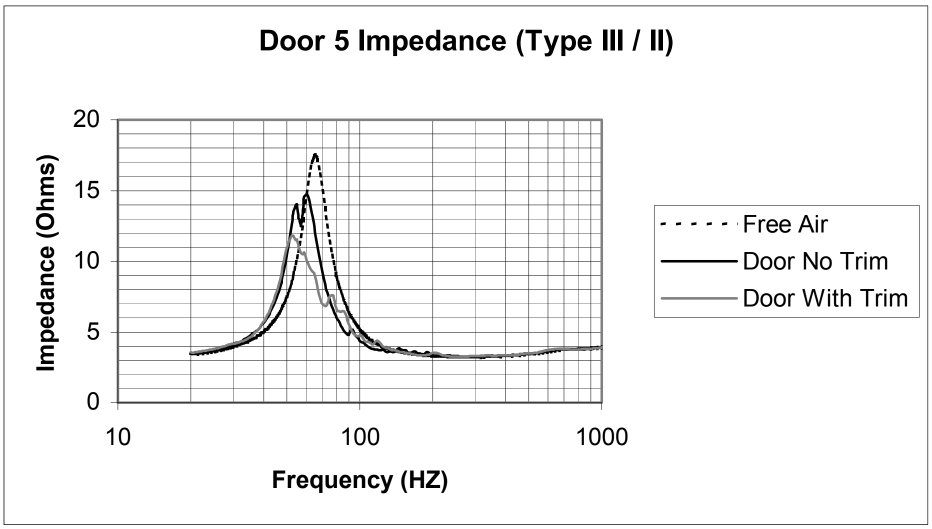
Type III Door Impedance: Complete resonance shift down in frequency



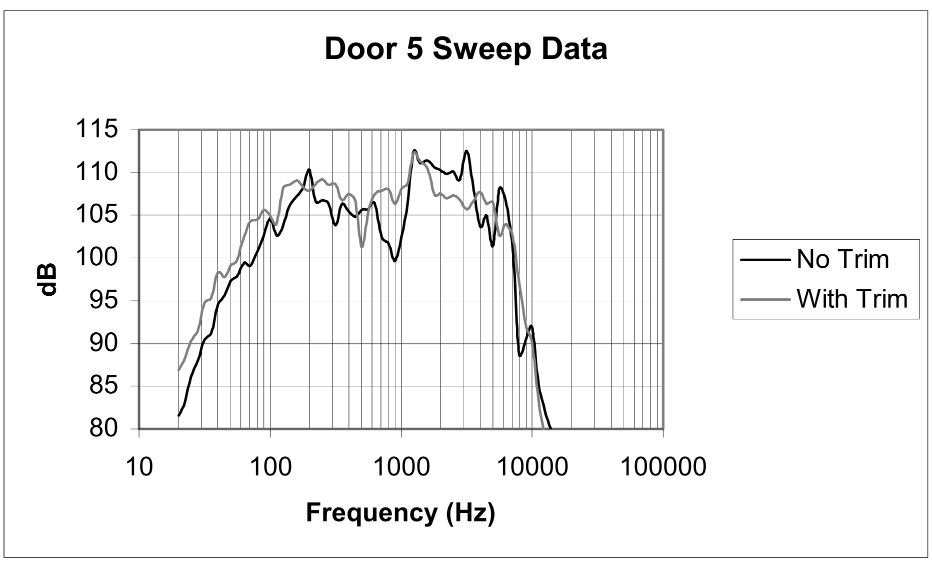
APPENDIX 1 (CONT.) DOOR 5

Door Type: Medium-Sized SUV

Door 5	Free Air	Door	With Trim
F	65.48	60.40	52.60
Q_m	4.41	3.98	2.18
Q_e	0.99	1.13	0.83
Q_t	0.81	0.88	0.60
V_D (liter)	0.04		
M_{ms}	10.08	12.56	
α		0.05	-0.33



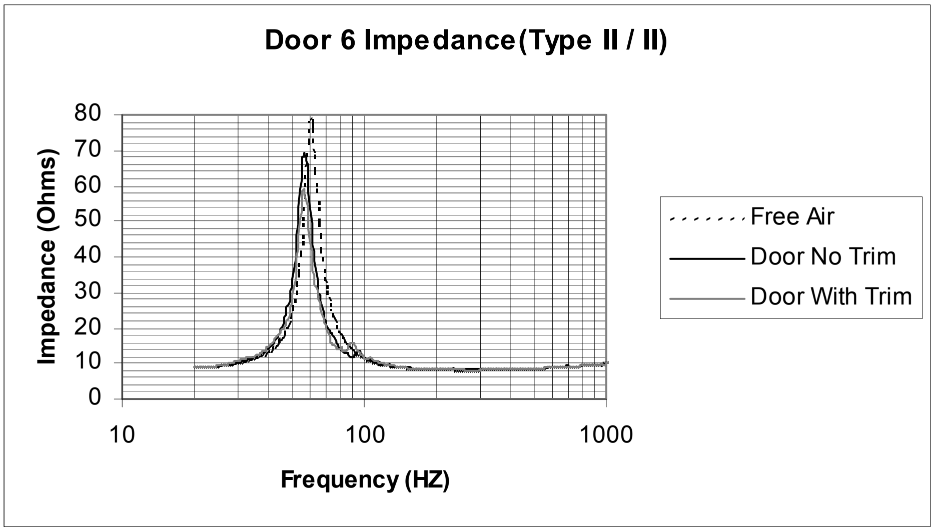
Type II Door Impedance: Skewing of the curve becoming asymptotic to the free air curves below resonance



APPENDIX 1 (CONT.) DOOR 6

Door Type: Coupe

Door 6	Free Air	Door	With Trim
F	59.70	57.03	57.03
Q_m	8.29	7.85	7.99
Q_e	0.96	1.05	1.27
Q_t	0.86	0.93	1.10
V_D (liter)	0.07		
α		0.05	0.27

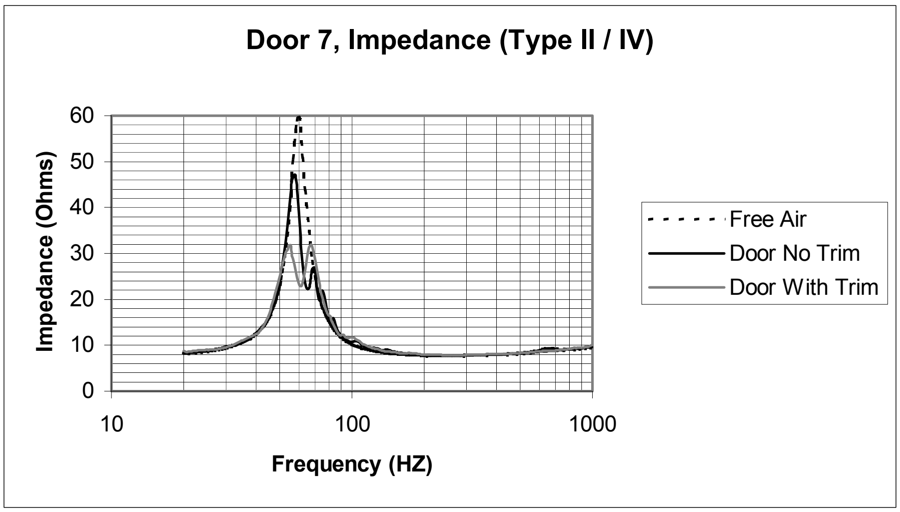


Type III Door Impedance: A complete resonance shift down in frequency

APPENDIX 1 (CONT.) DOOR 7

Door Type: Large Sedan

Door 7	Free Air	Door	With Trim
F	60.40	57.68	67.78
Q_m	7.52	6.72	7.43
Q_e	1.11	1.33	1.10
Q_t	0.97	1.11	0.96
V_D (liter)	0.07		
α		0.14	0.04
Q_L			7.80

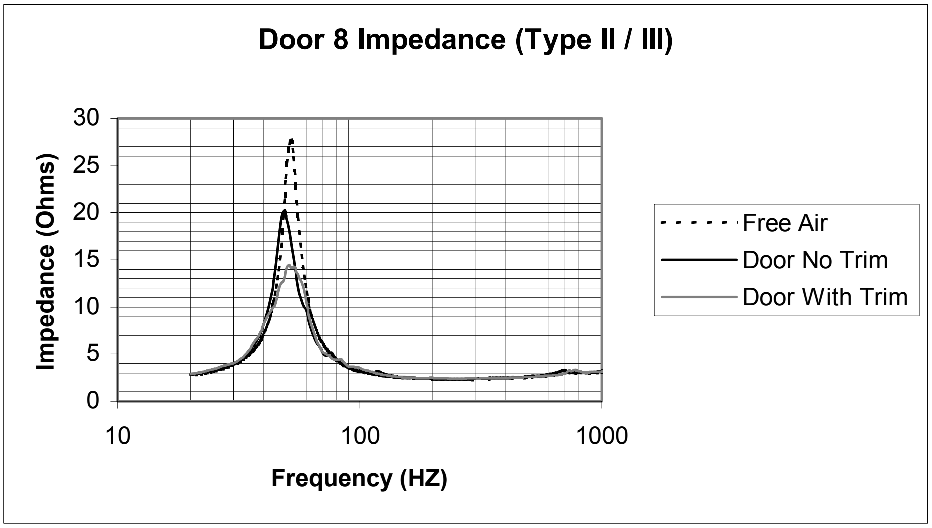


Type II Door Impedance: Skewing of the curve becoming asymptotic to the free air curves below resonance

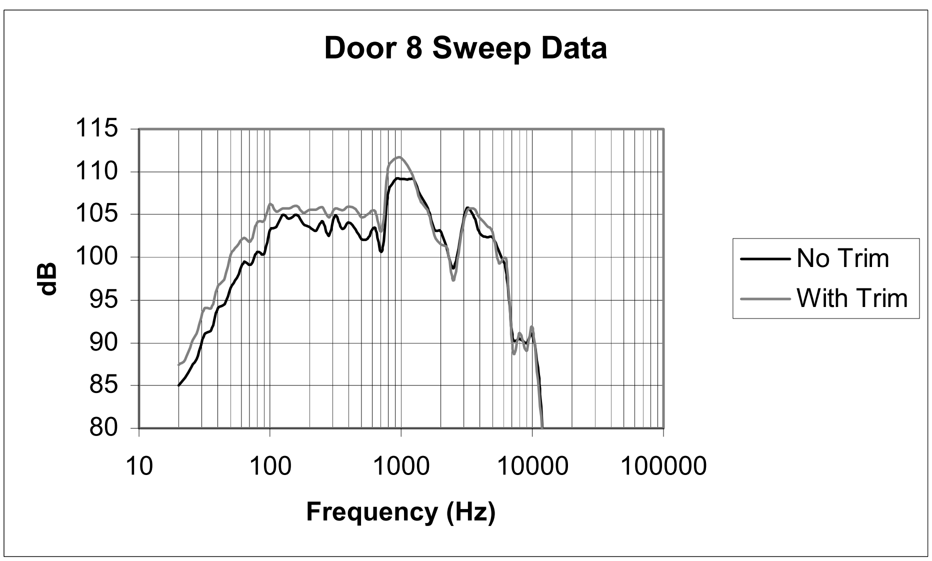
APPENDIX 1 (CONT.) DOOR 8

Door Type: Medium-Sized SUV

Door 8	Free Air	Door	With Trim
F	52.00	49.10	51.40
Q_m	7.43	5.50	3.89
Q_e	0.68	0.72	0.77
Q_t	0.62	0.64	0.64
V_D (liter)	0.09		
α		0.01	0.12



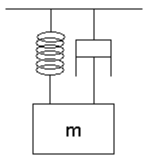
Type II Door Impedance: Skewing of the curve becoming asymptotic to the free air curves below resonance



10. APPENDIX 2

Transducer Simulation Model Equation

The mechanical end of the loudspeaker system is shown schematically below as a spring-mass-damper system driven by a voice coil:



To describe the system quantitatively, a differential equation can be used. Sum up the forces on the mass element and set that equal to the mass times the acceleration. The result after rearranging is:

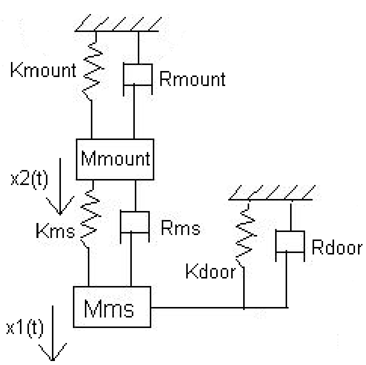
$$M_{ms}\frac{d^2x}{dt^2}+R_{ms}\frac{dx}{dt}+K_{ms}x=BlI(t) \tag{1}$$

However, the current has some interdependency on the voice coil velocity, and thus a second equation, one that describes the circuit relations for the electrical system is needed. That equation is:

$$L_e\frac{dI}{dt}+R_eI=V-Bl\frac{dx}{dt} \tag{2}$$

These two equations will be used for the transducer, and modified if necessary.

System Equations for Simulation Model



The first system equation will be established by summing up the forces acting on the moving mass of the driver and setting it equal to the moving mass times the acceleration of the moving mass: First is the mechanical stiffness force, which is proportional to the compression of the spring, (x1-x2), and is forced in the opposite direction of the compression. The second force is the mechanical resistance of the driver; it is treated the same way, except the force is proportional to the differences in the velocities of the two mass elements as opposed to the displacements.

The third force is the acoustical stiffness, and it is the pressure applied due to the stiffness multiplied by the surface area of the cone. The pressure applied, however, is the volume displacement - the cone area times the cone displacement multiplied by the acoustical stiffness. Hence, the factor of cone area squared. The other acoustical force is the acoustical resistance, which is proportional to the volume velocity of the cone. The last force entering into the equation is the electrical force from the motor.

$$M_{ms}\frac{d^2x_1}{dt^2} = -K_{ms}\left(x_1 - x_2\right) - R_{ms}\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) - S_d K_{door}\left(S_d x_1\right) - S_d R_{door}\left(S_d \frac{dx_1}{dt}\right) + BlI\left(t\right) \tag{3}$$

Rearranging this to a nicer form:

$$M_{ms}\frac{d^2x_1}{dt^2} + \left(S_d^2 R_{door} + R_{ms}\right)\frac{dx_1}{dt} + \left(S_d^2 K_{door} + K_{ms}\right)x_1 - R_{ms}\frac{dx_2}{dt} - K_{ms}x_2 = BlI\left(t\right) \tag{4}$$

For mass element number two, a similar method is used. The mass of the element times its acceleration is set equal to the forces acting on it. The first force is the stiffness of the door, the second the resistance of the door, the third the stiffness of the driver's suspension, and the fourth the resistance of the driver's suspension.

$$M_{mount}\frac{d^2x_2}{dt^2} = -K_{mount}x_2 - R_{mount}\frac{dx_2}{dt} + K_{ms}\left(x_1 - x_2\right) + R_{ms}\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) \tag{5}$$

And rearranging:

$$-R_{ms}\frac{dx_1}{dt} - K_{ms}x_1 + M_{mount}\frac{d^2x_2}{dt^2} + \left(R_{mount} + R_{ms}\right)\frac{dx_2}{dt} + \left(K_{mount} + K_{ms}\right)x_2 = 0 \tag{6}$$

The third relation is just that of the electrical system of the driver from before:

$$L_e\frac{dI}{dt} + R_e I = V - Bl\frac{dx_1}{dt} \tag{1}$$

Analysis

To analyze the system, we will use the Laplace transform to bring the displacements, voice coil current, and applied voltage into the complex frequency domain. Then we will get the transfer function of voltage. After transforming the equations, the relations in the s-domain are as follows:

$$\begin{gathered}
\left[M_{ms}s^2 + \left(S_d^2 R_{door} + R_{ms}\right)s + \left(S_d^2 K_{door} + K_{ms}\right)\right]X_1\left(s\right) - \left(R_{ms}s + K_{ms}\right)X_2\left(s\right) = BlI\left(s\right) \\
-\left(R_{ms}s + K_{ms}\right)X_1\left(s\right) + \left[M_{mount}s^2 + \left(R_{mount} + R_{ms}\right)s + \left(K_{mount} + K_{ms}\right)\right]X_2\left(s\right) = 0 \\
\left(L_e s + R_e\right)I\left(s\right) = V_{in}\left(s\right) - BlsX_1\left(s\right)
\end{gathered} \tag{7}$$

Solving these equations for the two displacements and then the impedance (1/current):

$$\frac{X_1(s)}{V_{in}(s)}=\frac{Bl}{(L_e s+R_e)\left\{M_{ms}s^2+\left(S_d^2R_{door}+R_{ms}+\frac{Bl^2}{(L_e s+R_e)}\right)s+S_d^2K_{door}+K_{ms}-\frac{(R_{ms}s+K_{ms})^2}{\left[M_{mount}s^2+(R_{mount}+R_{ms})s+(K_{mount}+K_{ms})\right]}\right\}} \tag{8}$$

$$\frac{X_2(s)}{V_{in}(s)}=\frac{Bl(R_{ms}s+K_{ms})}{(L_e s+R_e)\left\{\left[M_{ms}s^2+\left(S_d^2R_{door}+R_{ms}+\frac{Bl^2}{(L_e s+R_e)}\right)s+S_d^2K_{door}+K_{ms}\right]\left[M_{mount}s^2+(R_{mount}+R_{ms})s+(K_{mount}+K_{ms})\right]-(R_{ms}s+K_{ms})^2\right\}} \tag{9}$$

$$Z(s)=\frac{V_{in}(s)}{I(s)}=R_e+L_e s+\frac{Bl^2s}{M_{ms}s^2+\left(S_d^2R_{door}+R_{ms}\right)s+S_d^2K_{door}+K_{ms}-\frac{(R_{ms}s+K_{ms})^2}{M_{mount}s^2+(R_{mount}+R_{ms})s+(K_{mount}+K_{ms})}} \tag{10}$$

The impedance is of primary concern and will be converted to steady state:

$$Z(\omega)=R_e+i\omega L_e+\frac{iBl^2\omega}{-M_{ms}\omega^2+i\left(S_d^2R_{door}+R_{ms}\right)\omega+S_d^2K_{door}+K_{ms}+\frac{R_{ms}^2\omega^2-i2R_{ms}K_{ms}\omega-K_{ms}^2}{M_{mount}\omega^2-i(R_{mount}+R_{ms})\omega-(K_{mount}+K_{ms})}} \tag{11}$$